Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies

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In the study of analytical model improvement using measured vibration modes, there often exists an incompatibility between the analytical model of a vibrating structure and the corresponding measured vibration modes. Unless this inconsistency is resolved, the analytical model improvement exercise cannot be carried out properly. Generally, there are two possibilities: one is to condense the analytical model to match the measured modes, using a Guyan reduction or similar technique, and the other is to expand the measured modes to the same coordinate set as used by the analytical model. It is shown in this paper that condensing the analytical model tends to change the location of errors existing in the model and, thus, to jeopardize the error localization attempt, which is vital for model improvement practice. The other approach—expanding the measured modes—is shown to be feasible and effective for the error localization process. Furthermore, the paper proposes a method to deal with the case when the measured modes are complex due to a significant degree of damping. It is shown that both the errors in the analytical model and the spatial damping distribution can be located using expanded complex modes.

Nomenclature

m	= number of measured modes
N	= dimension of the analytical model
n	= number of coordinates employed by the measured modes
p	$= N - n$
$®(A)$	= rearrange the coordinates of vector A to move masters to the upper part
$®^{-1}(A)$	= rearrange the coordinates of vector A to the analytical modes order
$[H] \in R^{N \times N}$	= structural damping matrix
$[\underline{K}_a] \in R^{N \times N}$	= original analytical stiffness matrix
$[\underline{M}_a] \in R^{N \times N}$	= original analytical mass matrix
$[K_a] \in R^{N \times N}$	= analytical stiffness matrix after coordinate rearrangement $[K_a] = ®[\underline{K}_a]$
$[M_a] \in R^{N \times N}$	= analytical mass matrix after coordinate rearrangement $[M_a] = ®[\underline{M}_a]$
$[K_a]_R \in R^{n \times n}$	= Guyan reduced analytical stiffness matrix
$[M_a]_R \in R^{n \times n}$	= Guyan reduced analytical mass matrix
$[\underline{\phi}_a] \in R^{N \times N}$	= analytical mode shape matrix deduced from $[\underline{M}_a]$ and $[\underline{K}_a]$
$[\phi_x] \in R^{n \times m}$	= experimental mode shape matrix
$[\cdot\ \omega_a^2\ .] \in R^{m \times m}$	= diagonal analytical natural frequency matrix
$[\cdot\ \omega_x^2\ .] \in R^{m \times m}$	= diagonal experimental natural frequency matrix
$[\cdot\ \omega_c^2\ .] \in C^{m \times m}$	= diagonal experimental natural frequency matrix
$\{\varphi_{x1}\}_r \in R^{n \times 1}$	= rth measured mode
$\{\varphi_{x2}\}_r \in R^{p \times 1}$	= submode to be expanded for rth measured mode
$\{\varphi_x\}_r \in R^{N \times 1}$	= expanded rth measured mode, defined as $[(\varphi_{x1})_r^T \mid (\varphi_{x2})_r^T]^T$
$\{\underline{\varphi}_x\}_r \in R^{N \times 1}$	= expanded rth measured mode $(\varphi_x)_r = ®(\underline{\varphi}_x)_r$
$\{\varphi_{c1}\}_r \in C^{n \times 1}$	= rth measured mode
$\{\varphi_{c2}\}_r \in C^{p \times 1}$	= submode to be expanded for rth measured mode
$\{\varphi_c\}_r \in C^{N \times 1}$	= expanded rth measured mode, defined as $[(\varphi_{c1})_r^T \mid (\varphi_{c2})_r^T]^T$
$\{\underline{\varphi}_c\}_r \in C^{N \times 1}$	= expanded rth measured mode $(\varphi_c)_r = ®(\underline{\varphi}_c)_r$
$[\Delta K] \in R^{N \times N}$	= stiffness error matrix $[=(K_x) - (K_a)]$
$[\Delta K_c] \in C^{N \times N}$	= stiffness error matrix $[=(K_c) - (K_a) = (\Delta K) + (H)i]$

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Introduction

As current vibration practice demands more and more realistic mathematical models for dynamic structures, analytical model improvement using modal test results has become a major aspect in vibration research. The main target for this study is to modify an existing analytical model—generally obtained from finite element (FE) modeling—by correlating it with the (incomplete) measured data. Common efforts in analytical model improvement studies have been to optimize the analytical model using the measured modes so that the adjusted model agrees with the measured modes,[1,2] in spite of the fact that this may considerably modify the structural connectivity that the structure should obey and that the original analytical model correctly exhibited. Instead of correcting the whole analytical model as before, later efforts sought to localize the errors in the analytical model, using an incomplete set of measured modes.[3,4] These methods have been shown to rely on the availability of a rather large number of measured modes—a condition that is difficult to satisfy in practice.

Having found that introducing an iteration process and increasing computational costs cannot improve the violation of the structural connectivity and lead the model improvement process towards correct answers, a new method has been proposed[5] that seeks to locate the errors in the analytical model much more accurately by using a limited number of measured modes. It has also been shown that the iteration process can be effective in the model improvement process only when it is implemented on the errors that have first been located accurately.

The accurate location approach has also been developed for the simultaneous investigation of stiffness and damping properties of dynamic structures and analytical model improvements.[6] For this study, the measured complex modes are used directly (rather than seeking to extract the undamped modes from them, as many other efforts require[7–9]) in order to locate the errors in the analytical model and to locate the spatial damping elements.

It has been concluded that the key to both analytical model improvement and identification of the damping properties is to locate the errors in the model and the spatial damping elements, using the measured vibration modes. Unless this location is successful, the improved model and estimated damping matrix will not be physically sensible and any further attempt to iterate the process will generally result in a divergence.

Compatibility Inconsistency

One important issue that has not been fully discussed in previous studies is the practical incompatibility between the analytical model and measured (real or complex) modes in respect to the coordinates employed. Usually, an analytical model will employ a far greater number of coordinates to describe the vibration characteristics of a structure than is practical for the measured data. Two reasons for the experimental modal model to have a limited number of coordinates are 1) vibration measurement is too expensive to permit testing at many coordinates and 2) some coordinates may be either technically difficult to measure (such as rotation coordinates) or physically inaccessible (such as those coordinates inside the structure). Experience has shown that modal tests of typical structures may be limited to some 50–100 points, whereas the corresponding analytical models of a structure can be very detailed, and examples involving thousands of coordinates are not unusual in practice.

Since the techniques previously discussed can be implemented only when the analytical mass and stiffness matrices are compatible with the measured modes in terms of specified coordinates, model improvement and damping properties investigations cannot proceed until this incompatibility problem is resolved. Attempts to cope with this difficulty when the measured modes are supposed to be real are discussed below and, subsequently, the case of complex measured modes is dealt with. As indicated above, the key to both analytical model improvement and determination of damping properties is to locate the errors and/or damping elements in the model, and so attention will be focused here on the consequence of different approaches to this location study.

Current Efforts

Perhaps the most obvious way to overcome the incompatibility is to eliminate from the analytical model those coordinates not included in the measured modes. This can be done either by deleting the corresponding rows and columns of the analytical mass and stiffness matrices $[\underline{M}_a]$ and $[\underline{K}_a]$ and rederiving the analytical modes or by deleting the corresponding rows and columns on the analytical modes derived from the whole analytical model. However, both approaches result in major approximations and, hence, are rarely considered. In practice, two alternative strategies are possible for coping with this incompatibility and both are based upon the assumption that the measured vibration modes are real. The methods are 1) to condense the analytical model so that it is compatible with the measured modes or 2) to expand the measured modes to the full set of coordinates of the analytical model, possibly using the analytical model itself.

The problem of the compatibility of the measured modes and the analytical model was first dealt with by Guyan[10] who suggested that the mass matrix of a real structure can be reduced, as can the stiffness matrix, by eliminating the coordinates at which no external forces are applied. His purpose was to find an acceptable reduction technique for FE calculations, and he was not concerned with measurements. This matrix reduction method could then be employed in vibration studies to condense an analytical model to be compatible with measured data. More specifically, the original analytical mass and stiffness matrices (i.e., $[\underline{M}_a]$ and $[\underline{K}_a]$) can be partitioned into four submatrices, respectively, as follows:

$$[\underline{M}_a] = \begin{bmatrix} [M_a]_{11} & [M_a]_{12} \\ [M_a]_{21} & [M_a]_{22} \end{bmatrix} \qquad [\underline{K}_a] = \begin{bmatrix} [K_a]_{11} & [K_a]_{12} \\ [K_a]_{21} & [K_a]_{22} \end{bmatrix} \tag{1}$$

where $[M_a]_{11}$ and $[K_a]_{11}$ are $n \times n$ submatrices corresponding to the master coordinates (i.e., those tested experimentally). To be compatible with the measured modes, the analytical model is condensed so that the resultant model contains the following two matrices:

$$[K_a]_R = [K_a]_{11} - [K_a]_{12}[K_a]_{22}^{-1}[K_a]_{21} \tag{2}$$

and

$$[M_a]_R = [M_a]_{11} - [M_a]_{12}[K_a]_{22}^{-1}[K_a]_{21} - ([K_a]_{22}^{-1}[K_a]_{12})^T([M_a]_{21} - [M_a]_{22}[K_a]_{22}^{-1}[K_a]_{21}) \tag{3}$$

The mathematical consequence of this Guyan reduction is that the eigenproblem is closely but not exactly preserved. It is expected that the approximation would be inaccurate for higher frequency modes, and would be more inaccurate as the model is reduced to less DOFs.

An alternative approach seeks to expand the measured modes to the same coordinates as those of the analytical model. This exercise involves, first, rearranging the original matrices $[\underline{M}_a]$ and $[\underline{K}_a]$, as in Eq. (1). It is supposed that the expanded vibration mode shape of the rth measured mode $(\varphi_{x1})_r$ is denoted as $(\varphi_x)_r$, and, thus, that

$$\left(-\omega_r^2 \begin{bmatrix} [M_a]_{11} & [M_a]_{12} \\ [M_a]_{21} & [M_a]_{22} \end{bmatrix} + \begin{bmatrix} [K_a]_{11} & [K_a]_{12} \\ [K_a]_{21} & [K_a]_{22} \end{bmatrix}\right) \begin{Bmatrix} (\varphi_{x1})_r \\ (\varphi_{x2})_r \end{Bmatrix} = \{0\} \tag{4}$$

Thus, the unmeasured submode $(\varphi_{x2})_r$ of $(\varphi_x)_r$ can be expanded by using matrices $[\underline{M}_a]$ and $[\underline{K}_a]$ and the measured submode $(\varphi_{x1})_r$, such that

$$(\varphi_{x2})_r = -(-\omega_r^2[M_a]_{22} + [K_a]_{22})^{-1}(-\omega_r^2[M_a]_{21} + [K_a]_{21})(\varphi_1)_r \tag{5}$$

Once the submode $(\varphi_{x2})_r$ is obtained by Eq. (5), rearrangement of the coordinates in $(\varphi_x)_r$ should be performed to recover the order in the mode shapes corresponding to the original matrices $[\underline{M}_a]$ and $[\underline{K}_a]$:

$$Ⓡ^{-1}(\varphi_x)_r = (\varphi_x)_r \tag{6}$$

A measured mode thus expanded is effectively interpolated by the analytical model. If the analytical model contains errors, it can be expected that those interpolated coordinates in the expanded measured mode will not contain any information about the errors existing in the analytical model.

Discussion

The Guyan reduction was originally developed not for the location of errors in the analytical model of a vibrating structure, but for condensing the analytical model to an economical size so that the dynamic characteristics of the structure could be described in fewer coordinates by the

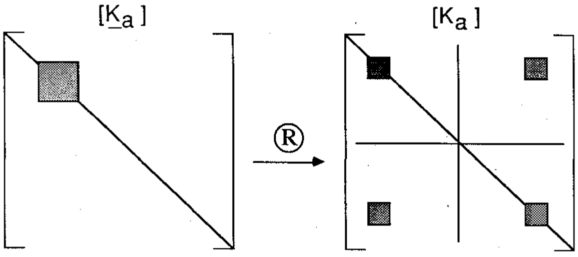

Fig. 1 An analytical stiffness matrix and matrix partition for the Guyan reduction.

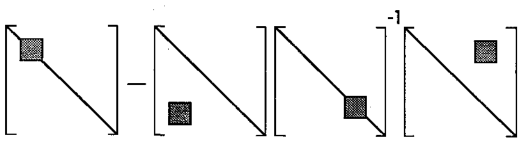

Fig. 2 Guyan reduction process for analytical stiffness matrix.

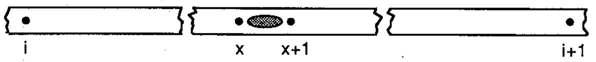

Fig. 3 A structure with coordinates x and $x+1$ being tested.

condensed model, which would reasonably possess similar natural frequencies and mode shapes of the structure. Improvements of this reduction process can also be found in recent literature.[11] Here again, attention is concentrated on the accuracy of the eigensolution of the condensed model. In order to use the Guyan reduction for the process of locating errors in the analytical model, it will be necessary to condense the analytical model down to those coordinates that were tested experimentally. However, the application of the approach to locate the errors[5] after this model condensation process has been applied may be severely impaired, because the errors existing in the analytical model will generally be scattered during the condensation process so that location of the errors in the analytical model may become very difficult or even not feasible. For instance, if an analytical stiffness matrix $[\underline{K}_a]$ contains modeling errors, then these errors may be scattered into all four of its partitioned submatrices in Eq. (1) by the reduction process, depending on the choice of the coordinates experimentally identified.

Figure 1 shows schematically an analytical stiffness matrix $[\underline{K}_a]$ with an erroneous region before reduction. In order to condense it into half its original size using the Guyan reduction approach, the matrix is partitioned by Eq. (1). If the measured and unmeasured coordinates alternate, (i.e., row/column r in matrix $[\underline{K}_a]$ corresponds to a measured coordinate, whereas row/column $r+1$ does not), then $[\underline{K}_a]$ will be partitioned in such a way that the rearranged stiffness matrix $[K_a]$ is as shown in Fig. 1. It can be seen that the erroneous elements are now scattered into all four submatrices. The condensed stiffness matrix $[K_a]_R$ achieved by Guyan reduction is shown in Fig. 2 and no longer exhibits precisely the original error location. This phenomenon can also be found in the literature,[12] but without the above interpretation.

It can be seen from the above that it would be unrealistic to expect a thus condensed analytical stiffness matrix $[K_a]_R$ still to hold the same location of the errors as the original $[K_a]$ does. It is also noticeable from Eq. (3) that the errors in $[K_a]_R$ will pollute the analytical mass matrix since the condensation of the mass matrix relies on $[K_a]_R$. In consequence, it is suggested that great care should be taken when the Guyan reduction is used for cases where the major concern is the location of errors in an analytical model using measured modes (and perhaps model improvement studies).

On the other hand, if the measured vibration mode $(\varphi_{x1})_r$ is expanded to the same coordinate set as the analytical model, as described above, then the possibility of scattering the errors in $[\underline{K}_a]$ is avoided, since this process does not change the connectivity in $[\underline{K}_a]$. It needs to be emphasized here that such a mode expansion approach does not change the measured modes at all; instead, it uses the analytical model to interpolate those coordinates that are not measured. Of course, it can be expected that modes thus obtained $[(\underline{\varphi}_x)_r]$ are not exactly the same as those modes actually measured at all the coordinates, since the measured modes are expanded by the analytical mass and stiffness matrices. (It is worth mentioning that all the measured coordinates are retained unaltered after the measured modes are expanded.) However, in an attempt to locate the errors in $[\underline{K}_a]$, which are to be pinpointed using those coordinates, location using the thus expanded measured modes could define the errors by those coordinates tested. Coordinates in the expanded $(\varphi_x)_r$ that were not tested will not possess any error location information. It is worthwhile to repeat that both the analytical mass, stiffness matrices, and the measured/expanded modes are rearranged before and after the mode expansion. Thus, modeling errors in $[\underline{K}_a]$ (not just $[K_{11}]$) can be located.

Suppose that coordinates i and j of a structure in Fig. 3 are two among the n measured coordinates, whereas in the analytical model, ten coordinates are specified between (and including) i and $i+1$. Furthermore, suppose that among these ten coordinates, modeling errors exist between coordinates x and $x+1$, and other possible modeling errors situated in other parts of the structure are unknown.

If model condensation is used to solve the coordinate incompatibility problem, then the dynamic characteristics of the structure defined by all ten coordinates will be concentrated into coordinates i and $i+1$ and, furthermore, coordinates i and $i+1$ will also contain similar information from all other coordinates of the structure due to the condensation process specified by Eqs. (1–3). Hence, the errors indicated by the location process, if any, between coordinates i and $i+1$ cannot be assigned simply to this location. (Clearly, there is no point in expecting any approach to locate precisely the mismodeled regions between coordinates x and $x+1$, because these two coordinates are not actually measured.) However, if the measured modes are expanded using $[\underline{M}_a]$ and $[\underline{K}_a]$, then the error location technique could be used to localize the error, which will be confined between coordinates i and $i+1$. In addition, there is no risk of the error being scattered in the condensed analytical model in this case, nor will other errors, if present, appear to be at this location due to model condensation.

Expansion of Measured Complex Modes

In the previous studies, only the undamped case has been considered. The vibration modes from measurement have effectively been supposed to be real so that they can be expanded using the analytical model. Such applications have been found in the literature,[13] albeit not for the purpose of error location. However, some vibrating structures are significantly damped, and the vibration modes identified experimentally will be considerably complex. In order to use such measured modes to locate errors in the analytical model and to study the damping properties of the structure, the measured complex modes have to be expanded in some way to the full coordinate set of the analytical model. Although there is no appropriate analytical damping matrix in existence (proportional damping is generally inappropriate for significantly damped structures), it is suggested here that the measured complex modes could still be expanded in a similar way as was used for the case of measured real modes, as outlined below.

The measured rth complex mode consists of the damped natural frequency $(\omega_c)_r$, damping loss factor η_r (for the sake of simplicity, the structural damping model is assumed in this

Table 1 Natural frequencies and damping loss factors

21 DOF System	Analytical model	Experimental model	
	Undamped	Damped	
Mode no.	frequency, Hz	Frequency, Hz	Damping loss factor
1	10.247	10.433	0.040673
2	26.615	26.974	0.079168
3	48.950	48.995	0.000944
4	65.561	66.574	0.095412
5	77.270	78.512	0.038553
6	94.436	94.401	0.005943
7	129.425	129.805	0.001260
8	154.283	156.197	0.006836
9	165.128	166.831	0.003896
10	184.810	184.808	0.001321
11	205.195	205.197	0.000040
12	221.929	221.932	0.000685
13	249.918	249.873	0.016287
14	269.443	279.794	0.000172
15	283.824	288.980	0.000297
16	290.790	290.928	0.000263
17	292.867	307.798	0.000001
18	330.097	330.097	0
19	371.159	388.123	0
20	471.907	471.907	0.000047
21	491.147	491.147	0.000153

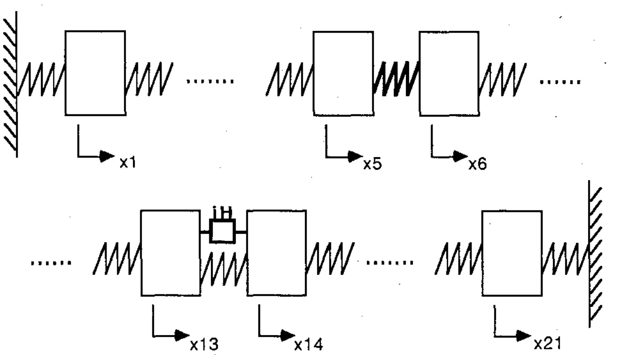

Fig. 4 A 21 DOF system with an incorrectly predicted stiffness component and an unpredicted hysteretic damper.

paper), and the complex mode shape $(\varphi_c)_r$. If the complex mode shape to be expanded is denoted as $(\varphi_c)_r$ and is

$$(\varphi_c)_r = [(\varphi_{c1})_r^T \mid (\varphi_{c2})_r^T]^T \tag{7}$$

then the usual eigenproblem described in Eq. (4) can be similarly partitioned into two parts, representing, respectively, the coordinates in the measured complex mode shape $(\varphi_{c1})_r$ and those remaining to be specified by means of the analytical model $[(\varphi_{c2})_r]$:

$$\left(-((\omega_c^2)_r(1+\eta_r i)) \begin{bmatrix} [M_a]_{11} & [M_a]_{12} \\ [M_a]_{21} & [M_a]_{22} \end{bmatrix} + \begin{bmatrix} [K_a]_{11} & [K_a]_{12} \\ [K_a]_{21} & [K_a]_{22} \end{bmatrix} \right) \times \begin{Bmatrix} \{\varphi_{c1}\}_r \\ \{\varphi_{c2}\}_r \end{Bmatrix} = \{0\} \tag{8}$$

The submode $(\varphi_{c2})_r$ can then be expanded by means of the analytical mass and stiffness matrices, and the measured mode $(\varphi_{c1})_r$:

$$(\varphi_{c2})_r = -\{-(\omega_c^2)_r(1+\eta_c i)[M_a]_{22} + [K_a]_{22}\}^{-1} \times \{-(\omega_c^2)_r(1+\eta i)[M_a]_{21} + [K_a]_{21}\}(\varphi_{c1})_r \tag{9}$$

Thus, the measured complex modes can be expanded to the full coordinate set so as to be compatible with the analytical model, based upon the analytical mass and stiffness matrices. Again, a coordinate rearrangement needs to be made in the expanded complex mode in Eq. (9) in order to resume the same order as the original analytical model:

$$\circledR^{-1}\{\varphi_c\}_r = \{\underline{\varphi}_c\}_r$$

Assessment of Different Approaches for Compatibility

The different approaches discussed and developed to solve the problem of incompatibility between the measured (real and/or complex) modes and the analytical vibration model of a structure are assessed in the following numerical studies. The system used here is the 21 degree-of-freedom system shown in Fig. 4. The full analytical stiffness and mass matrices ($[\underline{K}_a]$ and $[\underline{M}_a]$) can be formulated, as all the stiffness and mass elements are given and, hence, all the simulated real analytical vibration modes are known.

It is supposed that the analytical model of the system has two defects, one being that the stiffness component between coordinates 5 and 6 is underestimated by 20% and the other being that a hysteretic damper in between coordinates 13 and 14 is unmodeled. By considering these two defects, the experimental model of the system can be constructed and, thus, the "measured" complex modes can be computed for use in this study. Table 1 shows the natural frequencies and damping loss factors of all 21 modes for both models. The correct stiffness error matrix and the damping matrix of the system are shown in Fig. 5.

If a modal test were conducted on this system using all 21 coordinates, then the measured modes are defined in terms of the full coordinate set and so are completely compatible with the analytical model. Thus, there is no requirement either to

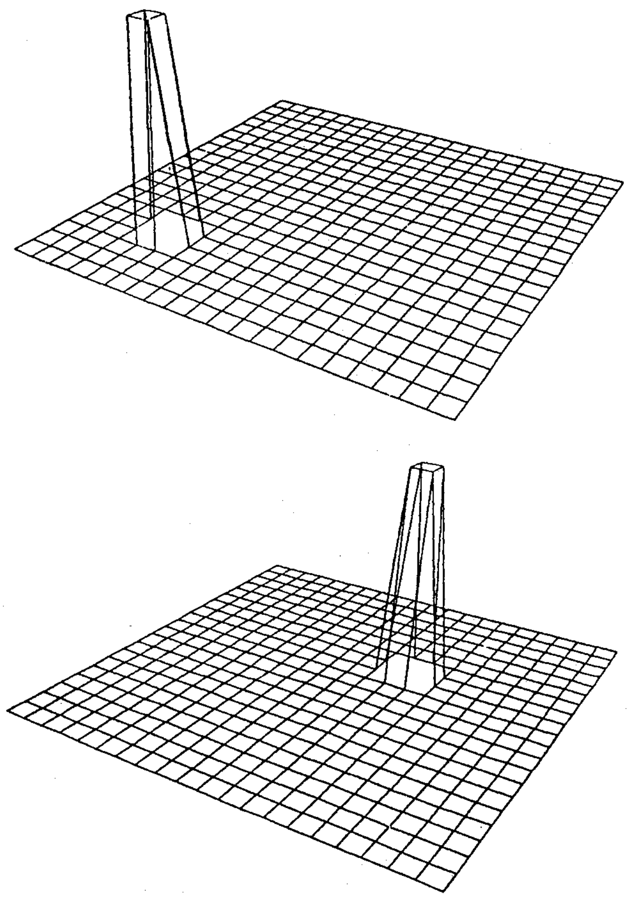

Fig. 5 The correct stiffness error matrix and damping matrix of the system shown in Fig. 4.

condense the model or to expand the modes, and the measured complex modes can be used to locate the stiffness errors in matrix $[\underline{K}_a]$ and the damping elements in matrix $[H]$. The method used to locate errors and the damper is proposed in the literature.[5,6] Basically, this method is to estimate the matrix product $[\Delta K]([\Phi_x][\Phi_x]^T)$ by calculating the right-hand side of the equation:

$$[\Delta K_c]([\phi_c][\phi_c]^T) = [\underline{M}_a][\phi_c][\cdot\ \omega_c^2\ \cdot][\phi_c]^T - [\underline{K}_a]([\phi_c][\phi_c]^T)$$

or

$$[\Delta K][\phi_c][\phi_c]^T + i[H][\phi_c][\phi_c]^T = [\underline{M}_c][\phi_c][\cdot\ \omega_c^2\ \cdot][\phi_c]^T - [\underline{K}_a][\phi_c][\phi_c]^T \qquad (10)$$

It is obvious that matrix product $[\Delta K][\phi_c][\phi_c]^T$ is an indicator of the stiffness modeling errors, although the error matrix $[\Delta K]$ cannot be calculated by Eq. (10) because of the rank deficiency of matrix product $[\phi_c][\phi_c]^T$.

Figure 6 shows the location results using Eq. (10) with each of modes 1, 2, or 3, individually. It can be seen from the results that both the stiffness error between coordinates 5 and 6 and the damper between coordinates 13 and 14 are located clearly, even with just a few of the modes.

In order to simulate the practical situation where the measured modes are defined at fewer coordinates than are used in the analytical model, only the coordinates with odd numbers in the complex modes are included, representing mode shapes which are experimentally identified. Then, these modes are expanded by Eq. (9) using matrices $[K_a]$ and $[M_a]$, and the thus expanded modes are then used to locate the stiffness errors in matrix $[\underline{K}_a]$ and the damping elements in matrix $[H]$. Figure 7 shows the location results using each of the expanded modes 1, 2, or 3, individually.

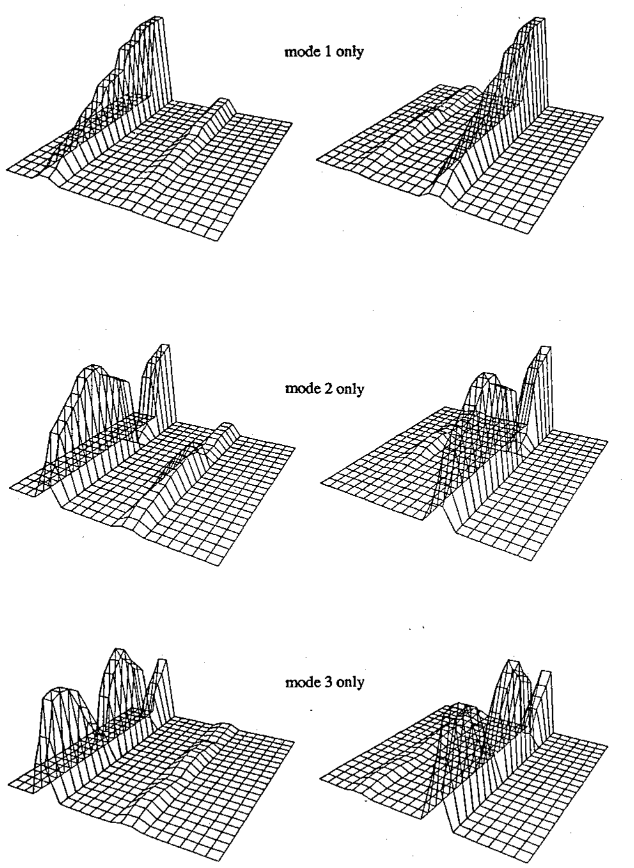

Fig. 6 Location of stiffness and damping elements in the system shown in Fig. 4 using experimental modes 1, 2 and 3, individually.

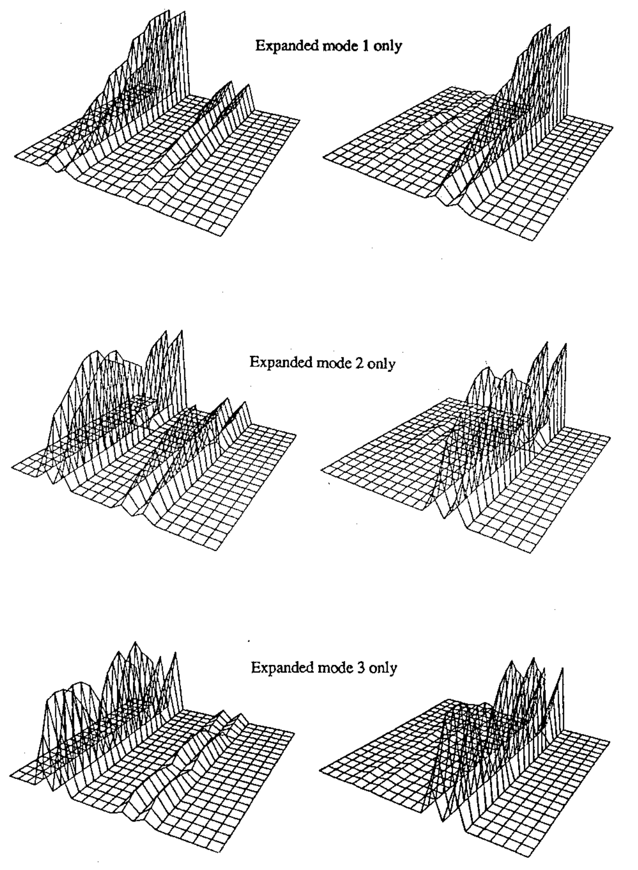

Fig. 7 Location of stiffness and damping elements in the system shown in Fig. 4 using expanded experimental modes 1, 2 and 3, individually.

Since the even-numbered coordinates in the expanded modes are interpolated by $[K_a]$ and $[M_a]$ (and effectively a zero damping matrix $[H]$), stiffness errors and damping elements will not be expected to show up in these coordinates. Therefore, the errors in matrix $[\underline{K}_a]$ should now be defined between coordinates 5 and 7 (instead of 6 and 7), these being the closest experimentally-identified coordinates. Indeed, the results using each expanded mode in Fig. 7 consistently indicate the stiffness errors between coordinates 5 and 7, with coordinate 6 exhibiting no stiffness modeling errors. The same argument for the location of the damper between coordinates 13 and 14 is also validated in Fig. 7. The results, using all these three expanded modes, consistently locate the damping element between coordinates 13 and 15 (instead of 13 and 14).

Concluding Remarks

In practice, a degree of incompatibility—in respect to the coordinates employed—always exists between the analytical model of a vibrating structure and the vibration modes that are identified experimentally. If this incompatibility problem is not resolved, then little further use can be made of the measured vibration modes to locate the errors in the analytical model and/or to study the damping properties.

Basically, there are two approaches to resolve this incompatibility: condensing the analytical model to those coordinates that are or that can be identified experimentally, and expanding the measured vibration modes to the full set of coordinates using the existing analytical model. It has been shown that, as far as the location of the modeling errors and/or damping components is concerned, a model condensation approach can be quite vulnerable, since the modeling errors in the analytical model will probably be scattered during the condensation process and, thus, jeopardize the location effort.

In order to preserve the correct location of modeling errors in the analytical model, it is suggested in this paper that the measured vibration modes be expanded on the basis of the analytical model and that the thus expanded modes be used to locate the modeling errors. The precision and advantage of such error location has been demonstrated. Furthermore, a complex mode expansion approach is proposed in order to deal with the situation when the measured modes are complex. It has been shown that thus expanded complex modes can be used effectively to locate both the stiffness modeling errors and the damping elements simultaneously. It is suggested by the results obtained that such a mode expansion technique can be used in practical modal studies.

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